Student Moving from Direct Modeling Strategies to Counting Strategies When Solving Word Problems Involving Groups of Ten

This story is a part of the series:

***What's Next?* Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

The students in this lesson were asked to solve two multiplication-grouping word problems and one measurement division word problem during an interview. Teachers sorted their work into categories according to strategies the students used. The teachers then developed a learning goal for these students on this day and a new set of problems designed to provide to students an opportunity to skip-count by twos, fives, and tens with understanding in the context of solving word problems.

Relevant Florida Mathematics Standards

MAFS.3.OA.1.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Background Information

In preparation for this lesson, consider reading chapter four of *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015), which provides a framework in which student strategies used to solve addition word problems are extended to multiplication word problems involving the grouping of countable objects.

The lesson places emphasis on *counting* strategies, especially skip-counting, and organizing objects in groups of ten. The article "Counting collections," by Schwerdtfeger and Chan (2007), offers a good introduction to the importance of guiding children's counting processes to create the basis for more sophisticated number sense, such as counting groups of ten to solve multiplication with base-ten concepts.

Carpenter, T. P, Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015) *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH. Heinemann.

Schwerdtfeger, J. K., & Chan, A. (2007). Counting collections. *Teaching Children Mathematics, 13*(7), 356–361.

Analyzing Student Thinking

To prepare for the lesson, the teacher provided the students with a formative assessment task. A group of teachers learning about student strategies interviewed the students individually. They were asked to solve three word problems. Two were multiplication problems, one with groups of six items and the other with groups of ten. The last was a measurement division problem in which the students were asked to find how many groups are needed:

A. I have 5 boxes with 6 cookies in each box. How many cookies do I have?

B. I have 8 boxes of pencils. There are 10 in each box. How many pencils do I have?

C. The second graders at Maplewood Elementary School raised $67 to buy books for the children's hospital. If each book cost $10, how many books can they buy?

Preparing for the interviews

Before conducting the interviews, the teachers anticipated the strategies students might use for each of the three problems. They asked themselves, "what might a third grader do?"

The interviewers were allowed the flexibility to change the context of the problems and to lower the numbers if they found the larger numbers to be too difficult for the individual student. For example, they might change $67 to $23 in problem C if the student could not successfully count 67 objects. They asked probing questions in attempt to learn more about students' thinking, but they did not lead or guide students to solve problems in any certain way.

The following charts (Figure 1) represent the various strategies teachers anticipated before the interviews that students would use. The first chart illustrates strategies students might use on

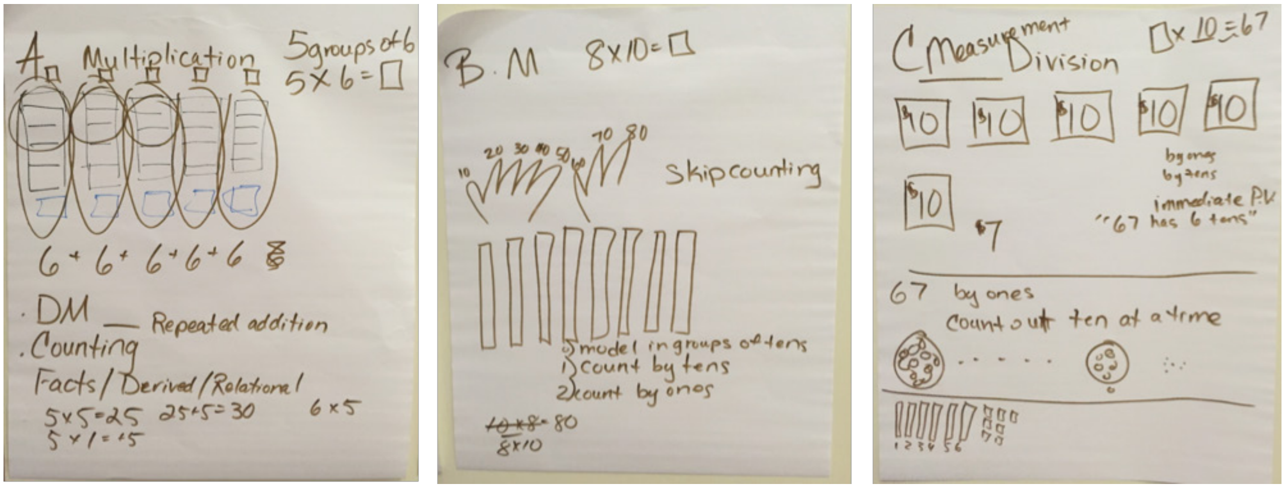

Figure 1. Teacher's anticipation of student strategies for interview problems.

Problem A. The picture on the left represents a *direct modeling* strategy where a student might create five groups of six linking cubes. Because the cubes were already linked in groups of five, students might keep them connected and add a sixth cube to each group. They might also use a different cube (top right of each group) to represent the boxes the cookies are in. They might also use repeated addition, skip-counting, or some known or derived facts.

The middle chart shows strategies they expected students to use to solve Problem B. These drawings represent strategies such as skip-counting using fingers to keep track of the number of counts, using base-ten rods and *counting by tens* or by ones or by some combination of tens and ones, and of simply knowing the multiplication fact directly.

For Problem C, the teachers anticipated students' using various ways to model six groups of ten. These included drawing boxes and writing ten in each one, drawing individual items and circling them in groups of ten, and using six base-ten rods and seven single cubes. They also anticipated that some students might use an abstract understanding of place value to know that "67 has six tens."

Students were informed that the purpose of the interviews was for the teachers to learn how children think about mathematics (which was exactly true). Each student was interviewed by two teachers who asked students to explain their thinking and took notes on the students' problem-solving process. The teachers read aloud one problem at a time and allowed the students significant time to think, model, write, or give and explain their answers. The students had linking cubes, base-ten blocks, and writing materials at their disposal and were encouraged to use whatever tools or method to solve the problem they desired.

After the students solved the three problems, the teachers sorted their thinking into groups according to the strategies they observed students using.

Names of strategies students use to solve problems involving groups of tens[1]

A student who uses a *direct modeling* strategy will represent each number in the problem using manipulatives (including fingers) or by drawing a model on paper. In this case, the child follows closely the structure of the story in the problem and models each item and the actions or relationship affecting these items. Children using this approach to solve multiplication problems by representing all the quantities in the problem and *counting by ones* to determine the solution.

1 The descriptions of strategies presented in this section are the current descriptions used by our team, and we consider them to be fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

For example, when solving Problem A (i.e., I have five boxes with six cookies in each box; how many cookies do I have?), an interviewed student drew six circles five separate times, and then counted the total number of circles, one by one, to determine the answer of 30 (see Figure 2). The interviewing teachers initially categorized this student work as a counting strategy (see bellow for description), because the student had written the number of objects in each group under each set. Upon revisiting the student's work, however, the teachers moved the student into the *direct modeling* group, because she counted each circle individually to find the total and did not make use of the sixes to skip count.

Students solve measurement division problems using *direct modeling* strategies by constructing the given number of sets, each containing the specified number of objects and then counting how many sets they created. For example, when solving a problem involving 12 objects to be divided into groups of four items, a student may start with the total number of objects in the problem, 12, which he or she then separates into groups of four, or may start forming groups of four while keeping track of the total number of objects used until reaching the total of 12 items.

In a variation of the previous strategy, the student who uses a *direct modeling, counting by numbers other than one* strategy represents all the quantities and counts by numbers other than one (such as tens and ones), keeping track with manipulatives or drawings (including fingers). For example a student drew eight boxes for Problem B and explained verbally that he counted by ten for each box while tapping each square with the pen to find the answer of 80 (Figure 2).

The student who uses a counting strategy recognizes that physically constructing the sets and counting each object are not necessary. For multiplication and division problems, counting strategies generally involve some form of skip-counting. For example, a student solved Problem A with smaller numbers (two groups of three items) by skip-counting three, six, nine and gave the answer nine, but when she solved the same problem with the original numbers (five groups of six), she reverted back to modeling the groups and counted by ones to find the answer. A counting strategy for a measurement division would involve skip-counting by the number of items in each group until reaching the total number of objects; the number of times the student skip-counted would be the answer. A student solving Problem C (the number of books that can be bought with $67) counted on by tens mentally, "10, 20, 30, 40, 50, 60," while keeping track on his fingers of how many numbers he said ("1, 2, 3, 4, 5, 6"). He realized that the $7 left from 60 to 67 were not enough to buy another book, so he concluded that the money was enough to buy six books.

The student who uses a *known fact* strategy recalls the answer from memory or responds that he or she "just knows" the answer. In problem B, for example, the student would know that eight groups of ten make 80 and would give the answer, often very quickly, and explain that he or she knew that fact.

Students who were unable to solve the problems by means of one of the strategies above were placed in the "couldn't solve" category. Some students who did not know where to start were also placed in this category. Students who used one of the aforementioned strategies but miscounted and found a close but incorrect answer were placed in the strategy group he or she used rather than under "couldn't solve".

Figure 2. Student's work for Problem A.

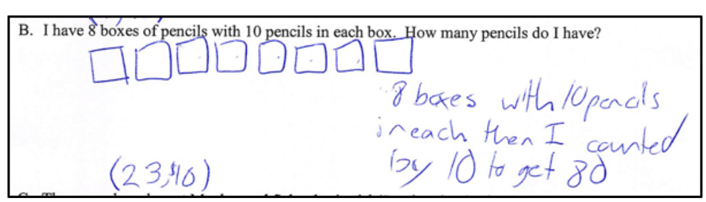

Figure 3. Student's work and explanation written down by interviewer

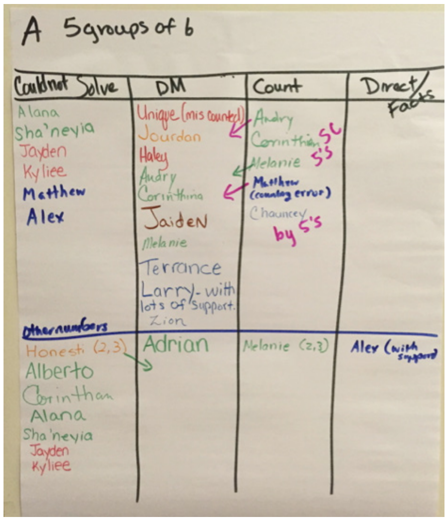

Figure 4. Student strategies for five groups of six.

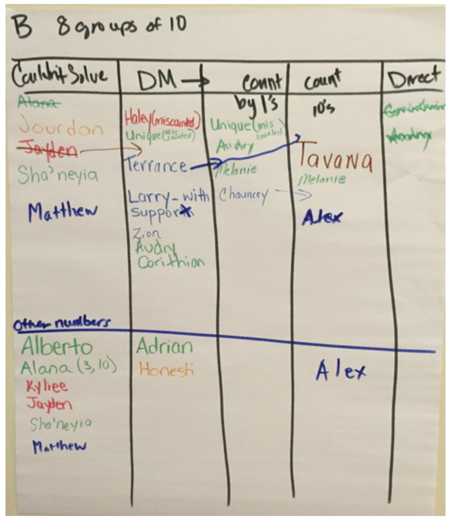

Figure 5. Student strategies for eight groups of ten.

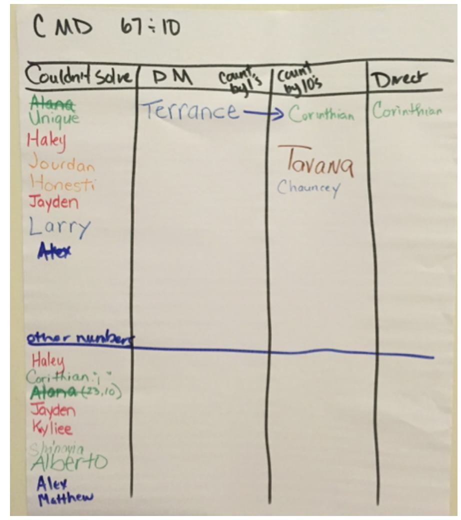

Figure 6. Student strategies for 67 divided by ten.

Strategies teachers observed these students using in the interviews

The teachers noticed that the multiplication problems were generally easier for students to solve than the measurement division problem. The multiplication problems also had a lot of variation in how students solved them. Most students were unable to solve the problems or using the simpler strategies (e.g., *direct modeling*, *counting by ones*), whereas few students used more sophisticated strategies such as *counting by tens*.

The teachers noticed two major groups of students: a group that had difficulty modeling multiplication problems and could not solve them, and one that could model the problem and had valid strategies for finding the answer. They wrote students names on sticky notes corresponding to whether they were in the former group (yellow) or the latter group (blue). (See Figure 7). During the subsequent lesson, teachers used these color-coded notes as nametags on students' desks for easy identification of these students. They suspected that the students in the yellow group might have had trouble understanding the context of the word problem and wanted to make sure they received additional support for comprehension of the story in the problem during the lesson.

Setting learning goals for students in this class on this day

The teachers decided to develop a lesson that addressed the needs of students in both the blue and the yellow groups with the intention of advancing the understanding of all of the students.

They wanted to help the students who were unable to solve the problems during the assessment (the yellow group) to develop viable strategies and acknowledged that the problem might need additional unpacking for them. To do this, they thought they should carefully create a problem with clear wording and a familiar context to which students could relate.

A focused discussion addressed how to help students who demonstrated they could model and count the objects by ones (the blue group) to develop more sophisticated and efficient problem-solving strategies. Skip-counting was identified as a useful skill they wanted the blue group to use with understanding. The teachers saw problems using groups of ten as a good way to get children to think about and work with objects in groups rather than single items. On this basis, the teachers developed the following learning goals.

1. *Students in the blue group will learn to skip-count by twos, fives, and tens with understanding.*

2. *Students in the yellow group will learn to create a pictorial representation for multiplication-grouping problems by representing groups of two, five, and ten objects.*

3. *Students will understand that a ten is composed of ten ones, and ten ones make a ten.*

4. *Students will use the context of the problem to communicate how to interpret the representation they use.*

Planning for the Lesson

The group of teachers created a mathematical problem that can be used to address each of the goals, as well as extensions of the problem for continued discussion during the lesson.

Problem: *I (the teacher) have four boxes of markers. There are 10 markers in each box. How many markers do I have in total?*

Extension: *Choose which one of these two questions you want to answer:*
— *How many markers are in 6 boxes?*
— *How many markers are in 12 boxes?*

Rationale for the problem selected

The teachers initially proposed the problem using pencil boxes. During further discussion, the teachers changed the objects to be boxes of markers, because they thought second graders would be more familiar with this scenario than with boxes of pencils. The number ten was chosen for the following three reasons.

1. It can lead to conversations and conceptual connections with place value ideas.

2. Students usually can skip-count verbally more easily by ten

The teachers noticed two major groups of students: a group that had difficulty modeling multiplication problems and could not solve them, and one that could model the problem and had valid strategies for finding the answer. They wrote students names on sticky notes corresponding to whether they were in the former group (yellow) or the latter group (blue). During the subsequent lesson, teachers used these color-coded notes as nametags on students' desks for easy identification of these students.

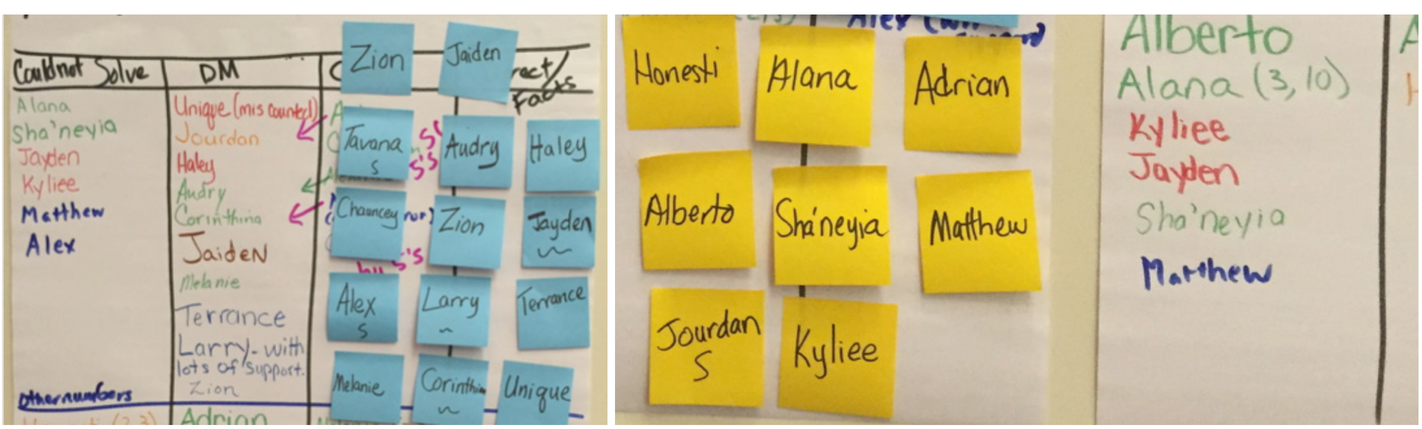

Figure 7. Students organized in two main target groups.

than by other numbers.

3. Ten is a landmark number.

Beginning with a small number of groups would allow all students to start thinking about groups of objects and reduce the number of instances of counting errors. The teachers planned to use the extension problem to encourage skip-counting by ten over *counting by ones*, which would become tedious for large numbers such as 60 or 120.

The main task was designed to allow students with a wide range of knowledge to be able to solve the problem. The students would have access to tools like connecting cubes, base-ten blocks, and writing materials during the lesson so each student could engage with numbers at whatever level of abstraction the individual students decided they needed at the time.

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Lesson Plan

The teachers planning the lesson the developed the following learning goals.

1. *Students in the blue group will learn to skip-count by twos, fives, and tens with understanding.*

2. *Students in the yellow group will learn to create a pictorial representation for multiplication-grouping problems by representing groups of two, five, and ten objects.*

3. *Students will understand that a ten is composed of ten ones, and ten ones make a ten.*

4. *Students will use the context of the problem to communicate how to interpret the representation they use.*

Rationale for strategic decisions in the lesson plan

The teachers planned to spend some time to introduce the scenario to the students and to ask students to repeat the problem back in their own words to ensure they understood what was happening in the story. Teachers presented the problems verbally and in writing. For the benefit of those students who did not seem to understand the problems posed during the interviews (i.e., the yellow group), the teachers planned to ask additional scaffolding questions about the number of markers that were in one box and the number of markers in two boxes.

The lesson was designed to allow time for the students to work individually. During this time, the teacher would circulate and ask scaffolding questions to encourage thinking about groups of ten rather than individual objects (for the blue group). The students who finished early would be given the extension problems to keep them engaged in mathematics at a higher level of challenge. The lesson would conclude with a whole-class discussion time meant to provide opportunities for students to explain their thinking to their peers, to listen to different strategies, and to draw connections among the various strategies students used.

Because the lesson was designed to be centered around a discussion of various student strategies and how they are related to each other, the teacher will watch for their strategies while the students are working individually. Students are expected to model four groups of ten objects and then to count the total number of objects by ones or by tens, to skip-count by tens four times, to use repeated addition (e.g., 10 + 10 + 10 + 10), or to know that four tens make 40.

The teacher will aim to notice whether each student is using the same strategies they used during the interviews by comparing their work with the sticky notes that were used to mark their desks. The teacher will also watch for students who skip-count by other numbers, such as two and five. The teacher will note how the students model the problem, whether they draw each object, or whether they use more abstract models. The teacher will also note whether the students break apart the linking cubes, which will be linked in groups of ten, or whether they use the connected rods. The teachers decided to organize the whole class discussion to present the different strategies in a deliberate sequence, ordering student solutions from least sophisticated to most sophisticated and selected strategies that use similar ideas with different representations. The teacher will choose one student to share while the students are working, and the teacher will ask those students whether they would be willing to share their thinking with the whole class (and warn them to be prepared).

Materials

After conducting the assessment and before launching the new problem, place sticky notes—color-coded for strategy—on the students' desks for ease of reference. Also place 50 cubes linked into groups of ten and 100+ base-ten blocks, both rods and singles, in front of each student. Notice that doing so provides enough linking cubes to model the initial task but not enough to model the extension task. This may encourage the more advanced students to move past simple *direct modeling* strategies, especially with single cubes.

Step-by-step plan

1. Introduce the context of the problem: Ask students about their familiarity with markers, whether they know where you can buy them, and how they come packaged. Make sure students understand that markers are usually bought in boxes with several markers in each box.

   Problem: I (the teacher) have four boxes of markers. There are 10 markers in each box. How many markers do I have in total?

2. Introduce the problem. Tell students you bought some boxes of markers. Each box contains ten markers, and each box contains the same number of markers If you bought four boxes of markers, how many markers are there in all? Ask several students to say back the story in their own words, focusing on members of the yellow group. Ask students to picture the boxes of markers in their minds, with their eyes closed.

3. Hand out copies of the problem on paper while continuing to ask students whether they can say back what's happening in the story problem. Make sure all members of the yellow group students can repeat in their own words the main idea: four boxes of markers, ten in each box. Ask students to think of a strategy to solve the problem. Once most show that they have a strategy in mind through a hand signal, ask students to start solving the prob-

lem. Remind students they can use whatever tools they want to solve it and whatever method they feel most comfortable with.

4. Allow enough time for students to solve the problem whichever way they choose and to record their thinking using numbers, pictures, or words.

5. While the students are working, circulate and notice and record what tools they use.

    a. For those who use linking cubes, notice who breaks the rods apart and who works with the connected rods.

    b. Notice also who is has difficulty getting started and point them back to the problem. In this case, ask scaffolding questions: How many markers are in one box? How many markers are in two boxes?

    c. Ask the students who have found an answer ask, "How did you get that?"

6. While the students work, monitor their progress and identify students using *direct modeling* with groups of ten, counting on, or any of the more advanced strategies.

7. Select one student using each type of strategy and ask these students whether they are willing to share their work with the whole class during discussion time.

8. At the end of the work time, direct the students' attention to the board. Have the selected students identified during work time share their strategies (i.e., strategies that focus on models using groups of ten).

9. Have one student who modeled and counted by ones draw his or her explanation on the board and explain what he or she did while the class listens (see Figure 8 for an example). Next, ask another student to repeat what that student said. Ask what the images in the student's strategy represent. For example, if the student drew circles to represent the markers,

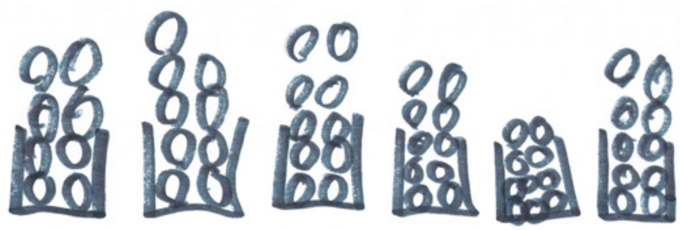

Figure 8. ***Direct modeling*** *and* ***counting by ones*** *strategy for the boxes-of-markers problem.*

Figure 9. ***Direct modeling*** *and* ***counting by tens*** *strategy for the boxes-of-markers problem.*

ask student to explain what each square represents, then ask what a group of squares represents. Ask what the answer represents.

10. Next, have a student who modeled and counted by tens share. For example, find a student who drew four bars and counted the, "10, 20, 30, 40," perhaps while touching each bar (e.g., in Figure 9, the student counted the group of ones as a ten even though he modeled with singles). Ask the student to explain why he knew to count by tens. You might may also ask how many markers would be in three boxes to determine whether the student can easily transfer knowledge to a new situation.

Reflection

The classification into blue and yellow groups eased the process of differentiating the discussion and scaffolding questions to meet the needs of different students. During the lesson, the teacher stopped by each member of the yellow group and asked the student to explain in his or her own words what the problem was asking. With members of the blue group, the teacher focused on asking strategy-related questions.

What we learned about the students

In the post-lesson discussion, the teachers noted that many students started working on the problem by breaking apart the connected rods even though they were already in groups of ten. The teachers found that, although the purpose of the rods was to illustrate the point that one ten is the same as ten ones, this concept was not readily taken up by the students. The teachers recognized the acceptability of *counting by ones* for students not yet ready to move on to grouping objects and manipulating the groups. On the other hand, teachers agreed they should continue to encourage students to move toward more advanced counting strategies by scaffolding that idea with concrete objects in groups of ten.

Most of the students who were not able to solve any of the problems in the interviews were able to produce a valid strategy for the main task. The teachers conjectured that reviewing what the problem says and having students retell it in their own words helped students to understand the problem.

The follow-up task served to help with classroom management. Using this device allowed the teacher to work with the students who needed more help. Having the choice between the two amounts also seemed to increase the students' motivation to engage with the task, perhaps because it gave them some control.

Ideas for the next lesson

The students were able to grasp the concept of grouping by tens during the lesson. Future lessons could include further exploration of grouping items by tens by means of measurement division problems.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**